Dana Center
Mathematics
PATHWAYS

Ensuring Rigor in First-Year Mathematics Courses

Connie Richardson, Manager, Higher Education Course Programs
Joan Zoellner, Course Program Specialist

December 7, 2018

Session Outcomes

Participants will:

- Explore the meaning of rigor in mathematics.
- Discuss ways to promote rigor in the first year mathematics/ quantitative reasoning courses.
- Engage with resources from the field and the professional associations.

Why are we exploring rigor?

These are some of the things we hear:

1) Concerns about whether it is realistic for students with weak math backgrounds to pass a rigorous college-level math course within their first year.

2) Questions about the curricular choices offered to students under math pathways (e.g. the belief that offering students statistics or quantitative reasoning, rather than a calculus-prep algebra course, is weakening the degree).

3) Speculation that offering stretch courses or support courses will lessen the rigor of the gateway math courses.

Concerns about rigor

Has the concern of rigor come up in department meetings? Enter the number of the concern in the chat box. If you choose #4, enter the reason.

1) Yes, but only as it relates to algebraic-intensive courses (Pre-Calc, College Algebra, etc.)
2) Yes, but only as it relates to non algebraic-intensive courses (Quantitative Reasoning, Statistics, etc.)
3) Yes, as it relates to offering stretch or support courses
4) Yes, but for another reason
5) No, concerns about rigor have not been raised.

Breakout #1: Sharing Thoughts About Rigor

Choose a reporter and a timekeeper.

In your breakout, discuss:

- *What concerns related to rigor have come up during department discussions?*
- *What interests you the most about the conversation around rigor?*
- *What motivated you to join this webinar?*

Share-out #1: Sharing Thoughts About Rigor

Reporter:

- *What concerns related to rigor have come up during department discussions?*
- *What interests you the most about the conversation around rigor?*
- *What motivated you to join this webinar?*

Other participants:

If there is anything else you want to share that has not been shared, please share in the chat box.

Dana Center's Understanding of Rigor

Rigor in mathematics is a set of skills that centers on the communication and use of mathematical language.

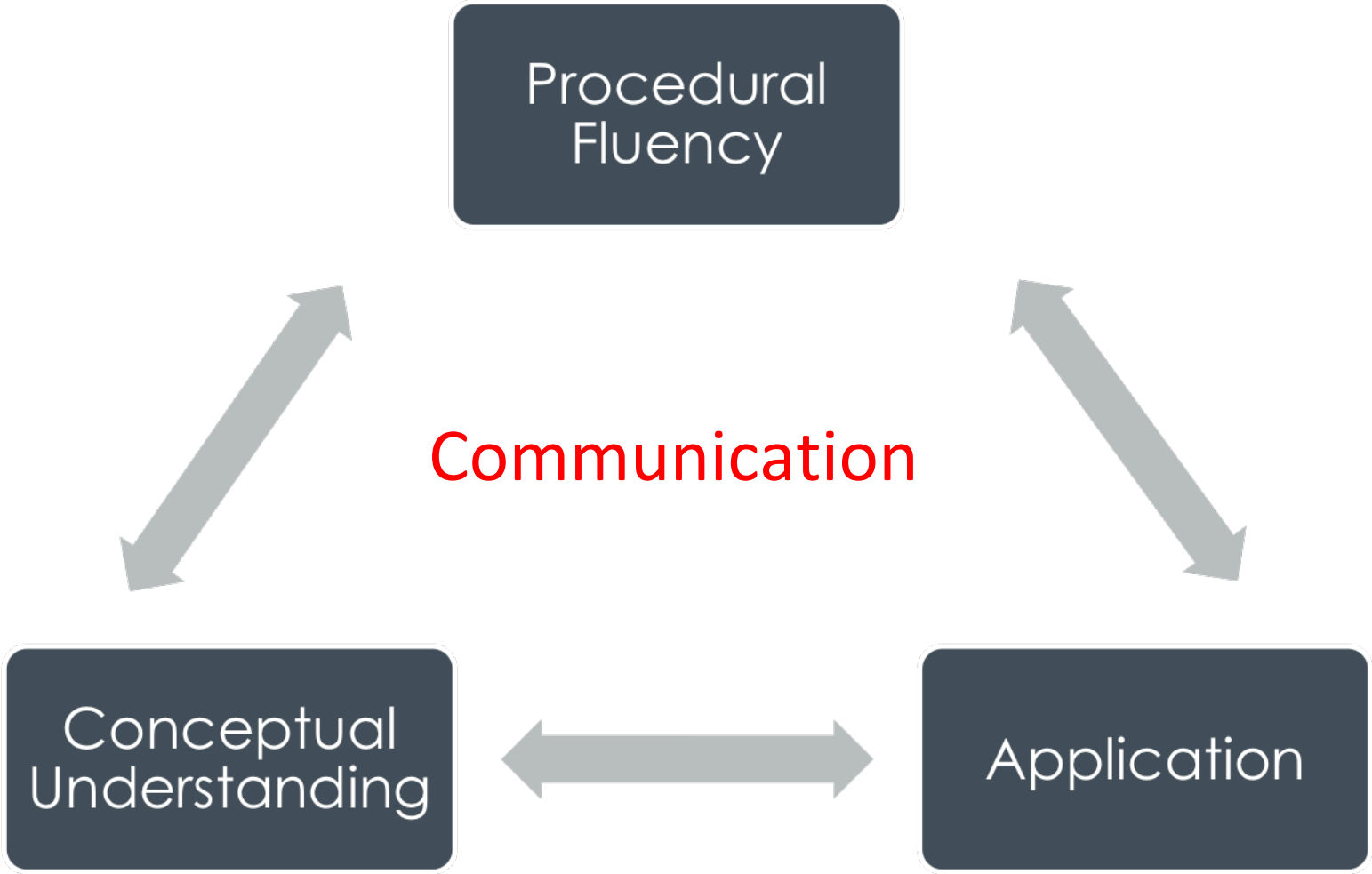
Towards a practical view of rigor

- We should attend to all of our math courses, whether it be statistics-, modeling- or algebra-based, to ensure that they are all taught with rigor.
- To learn mathematics, all students must have the opportunity to tackle rich problems and productively struggle with them.
- They must not only solve those problems but also be able to articulate the basis of an argument at a level of precision appropriate to the course.
- Math departments should play an essential role in determining the content of their introductory courses in conjunction with the views of the professional associations and the needs of the institution's various programs of study.

Classroom Tip ✓

If you provide an exam study guide, mix it up! Don’t put the problems in the order of the chapter, or all of the like problems together.

Components of Rigor



Classroom Tip ✓

Begin new concepts with an application problem.

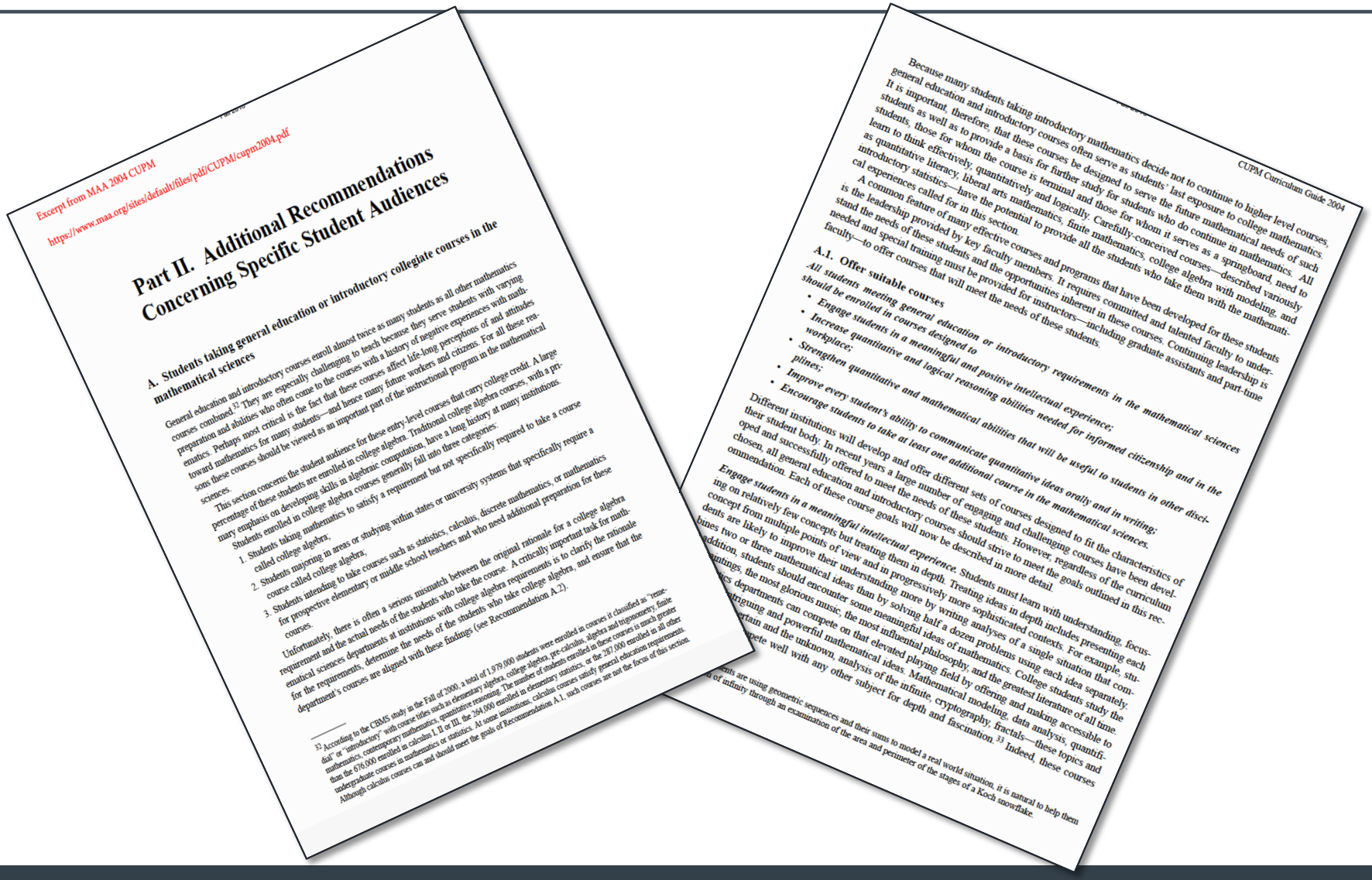
Wrap-up the problem by writing the answer in a complete sentence that thoroughly answers the question.

Any Pathway Can Be Rigorous

The 17 professional associations of mathematicians which comprise the CBMS have endorsed the idea that there are many areas of mathematics that, when well taught, can serve as appropriate introductions to college mathematics and mathematical thinking and work.

http://www.cbmsweb.org/

Excerpt from MAA 2004 CUPM



Excerpt from MAA 2004 CUPM

https://www.maa.org/sites/default/files/pdf/CUPM/cupm2004.pdf

Part II. Additional Recommendations Concerning Specific Student Audiences

A. Students taking general education or introductory collegiate courses in the mathematical sciences

General education and introductory courses enroll almost twice as many students as all other mathematics courses combined.[32] They are especially challenging to teach because they serve students with varying preparation and abilities who often come to the courses with a history of negative experiences with mathematics. Perhaps most critical is the fact that these courses affect life-long perceptions of and attitudes toward mathematics for many students—and hence many future workers and citizens. For all these reasons these courses should be viewed as an important part of the instructional program in the mathematical sciences.

This section concerns the student audience for these entry-level courses that carry college credit. A large percentage of these students are enrolled in college algebra. Traditional college algebra courses, with a primary emphasis on developing skills in algebraic computation, have a long history at many institutions.

Students enrolled in college algebra courses generally fall into three categories:

1. Students taking mathematics to satisfy a requirement but not specifically required to take a course called college algebra;
2. Students majoring in areas or studying within states or university systems that specifically require a course called college algebra;
3. Students intending to take courses such as statistics, calculus, discrete mathematics, or mathematics for prospective elementary or middle school teachers and who need additional preparation for these courses.

Unfortunately, there is often a serious mismatch between the original rationale for a college algebra requirement and the actual needs of the students who take the course. A critically important task for mathematical sciences departments at institutions with college algebra requirements is to clarify the rationale for the requirements, determine the needs of the students who take college algebra, and ensure that the department's courses are aligned with these findings (see Recommendation A.2).

[32] According to the CBMS study in the Fall of 2000, a total of 1,970,000 students were enrolled in courses it classified as "remedial" or "introductory" with course titles such as elementary algebra, college algebra, pre-calculus, algebra and trigonometry, finite mathematics, contemporary mathematics, quantitative reasoning. The number of students enrolled in these courses is much greater than the 676,000 enrolled in calculus I, II or III, the 264,900 enrolled in elementary statistics, or the 287,000 enrolled in all other undergraduate courses in mathematics or statistics. At some institutions, calculus courses satisfy general education requirements. Although calculus courses can and should meet the goals of Recommendation A.1, such courses are not the focus of this section.

CUPM Curriculum Guide 2004

Because many students taking introductory mathematics decide not to continue to higher level courses, general education and introductory courses often serve as students' last exposure to college mathematics. It is important, therefore, that these courses be designed to serve the future mathematical needs of such students as well as to provide a basis for further study for students who do continue in mathematics. All students, those for whom the course is terminal and those for whom it serves as a springboard, need to learn to think effectively, quantitatively and logically. Carefully-conceived courses—described variously as quantitative literacy, liberal arts mathematics, finite mathematics, college algebra with modeling, and introductory statistics—have the potential to provide all the students who take them with the mathematical experiences called for in this section.

A common feature of many effective courses and programs that have been developed for these students is the leadership provided by key faculty members. It requires committed and talented faculty to understand the needs of these students and the opportunities inherent in these courses. Continuing leadership is needed and special training must be provided for instructors—including graduate assistants and part-time faculty—to offer courses that will meet the needs of these students.

A.1. Offer suitable courses

All students meeting general education or introductory requirements in the mathematical sciences should be enrolled in courses designed to

- ***Engage students in a meaningful and positive intellectual experience;***
- ***Increase quantitative and logical reasoning abilities needed for informed citizenship and in the workplace;***
- ***Strengthen quantitative and mathematical abilities that will be useful to students in other disciplines;***
- ***Improve every student's ability to communicate quantitative ideas orally and in writing;***
- ***Encourage students to take at least one additional course in the mathematical sciences.***

Different institutions will develop and offer different sets of courses designed to fit the characteristics of their student body. In recent years a large number of engaging and challenging courses have been developed and successfully offered to meet the needs of these students. However, regardless of the curriculum chosen, all general education and introductory courses should strive to meet the goals outlined in this recommendation. Each of these course goals will now be described in more detail.

Engage students in a meaningful intellectual experience. Students must learn with understanding, focusing on relatively few concepts but treating them in depth. Treating ideas in depth includes presenting each concept from multiple points of view and in progressively more sophisticated contexts. For example, students are likely to improve their understanding more by writing analyses of a single situation that combines two or three mathematical ideas than by solving half a dozen problems using each idea separately. In addition, students should encounter some meaningful ideas of mathematics. College students study the paintings, the most glorious music, the most influential philosophy, and the greatest literature of all time. Mathematics departments can compete on that elevated playing field by offering and making accessible to students fascinating and powerful mathematical ideas. Mathematical modeling, data analysis, quantification and the unknown, analysis of the infinite, cryptography, fractals—these topics and courses can compete well with any other subject for depth and fascination.[33] Indeed, these courses

students are using geometric sequences and their sums to model a real world situation, it is natural to help them of infinity through an examination of the area and perimeter of the stages of a Koch snowflake.

https://www.maa.org/sites/default/files/pdf/CUPM/cupm2004.pdf

Breakout #2: MAA CUPM Article

Choose a reporter and a timekeeper.

In your breakout, discuss:

- *What information did you find that resonates with you?*

Share-out #2: MAA CUPM Article

Reporter:

- *What information did you find that resonates with you?*

Other participants:

If there is anything else you want to share that has not been shared, please share in the chat box.

Is the following a characteristic of a rigorous course?

Provide connections among mathematical ideas

Yes

No

Is the following a characteristic of a rigorous course?

Require memorization of rules and procedures and use of a rote procedure to solve problems

Yes

No

Is the following a characteristic of a rigorous course?

Teachers doing the work while students watch

Yes

No

Is the following a characteristic of a rigorous course?

Students know how to perform a list of algebraic tasks such as: multi-step factoring, rationalizing n^{th} roots, completing the square, etc.

Yes

No

Learning Experiences

Learning experiences that involve rigor …	Experiences that do not involve rigor …
challenge students	are more “difficult,” with no purpose (overly-complicated polynomial long division)
require effort and tenacity by students	require minimal effort
focus on quality (rich tasks)	focus on quantity (more pages to do)
include entry points and extensions for all students	are offered only to gifted students

https://www.nctm.org/News-and-Calendar/Messages-from-the-President/Archive/Linda-M_-Gojak/What_s-All-This-Talk-about-Rigor_/

Learning Experiences

Learning experiences that involve rigor …	Experiences that do not involve rigor …
provide connections among mathematical ideas	do not connect to other mathematical ideas
contain rich mathematics that is relevant to students	contain routine procedures with little relevance
develop strategic and flexible thinking	follow a rote procedure
encourage reasoning and sense making	require memorization of rules and procedures without understanding
expect students to be actively involved in their own learning	often involve teachers doing the work while students watch

https://www.nctm.org/News-and-Calendar/Messages-from-the-President/Archive/Linda-M_-Gojak/What_s-All-This-Talk-about-Rigor_/

Challenges with ensuring rigor

What are the biggest challenges to ensuring that your gateway math courses are rigorous?

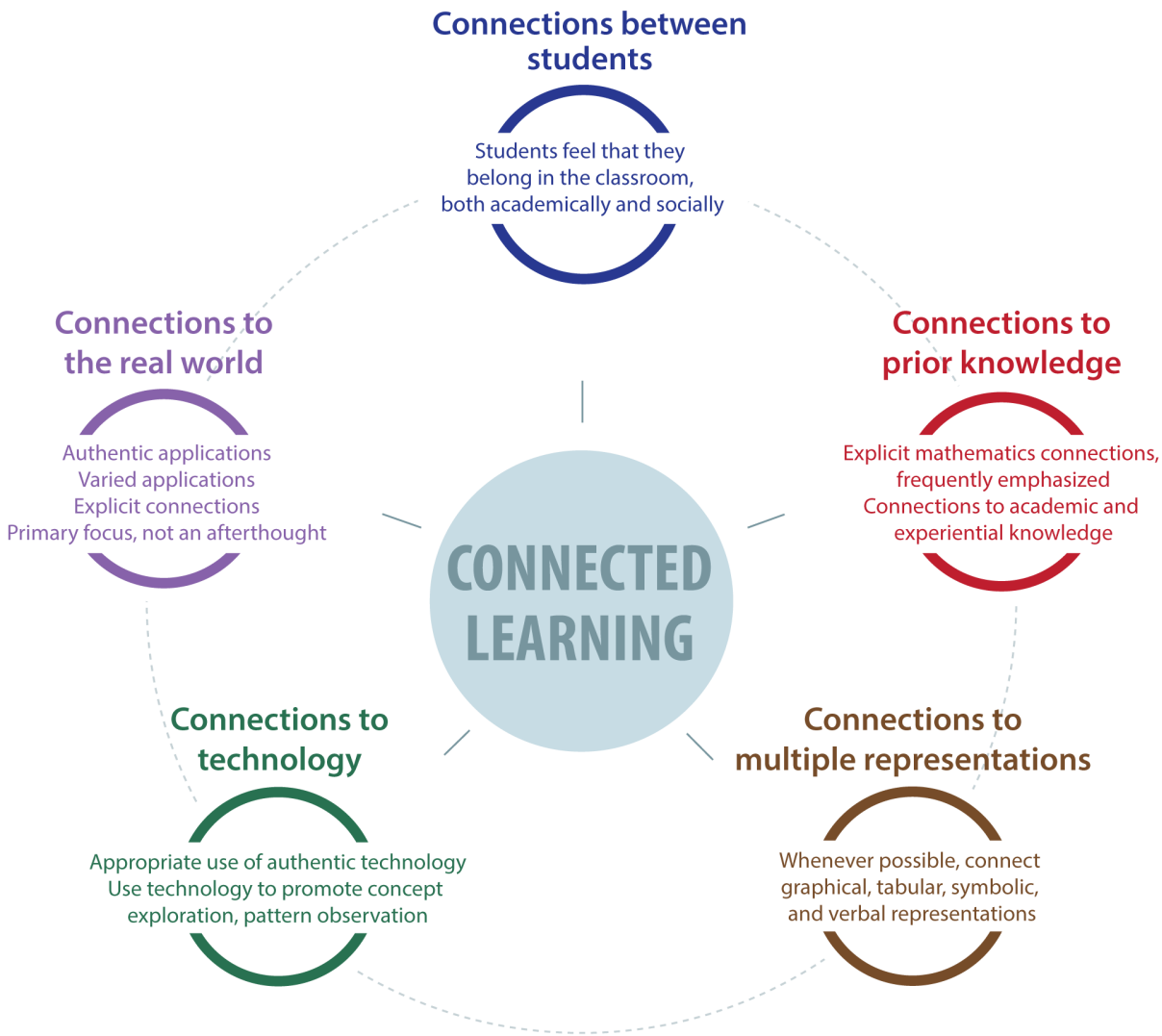
Enter the number of the challenge in the chat box. If you choose #5, enter the challenge in the chat.

1) Making explicit connections between concepts
2) Using relevant mathematical scenarios
3) Helping students develop strategies that make sense to them, rather than relying on memorization of rote procedures
4) Encouraging students to work actively and take control of their learning
5) Other

Activities and Assignments That Promote Rigor:

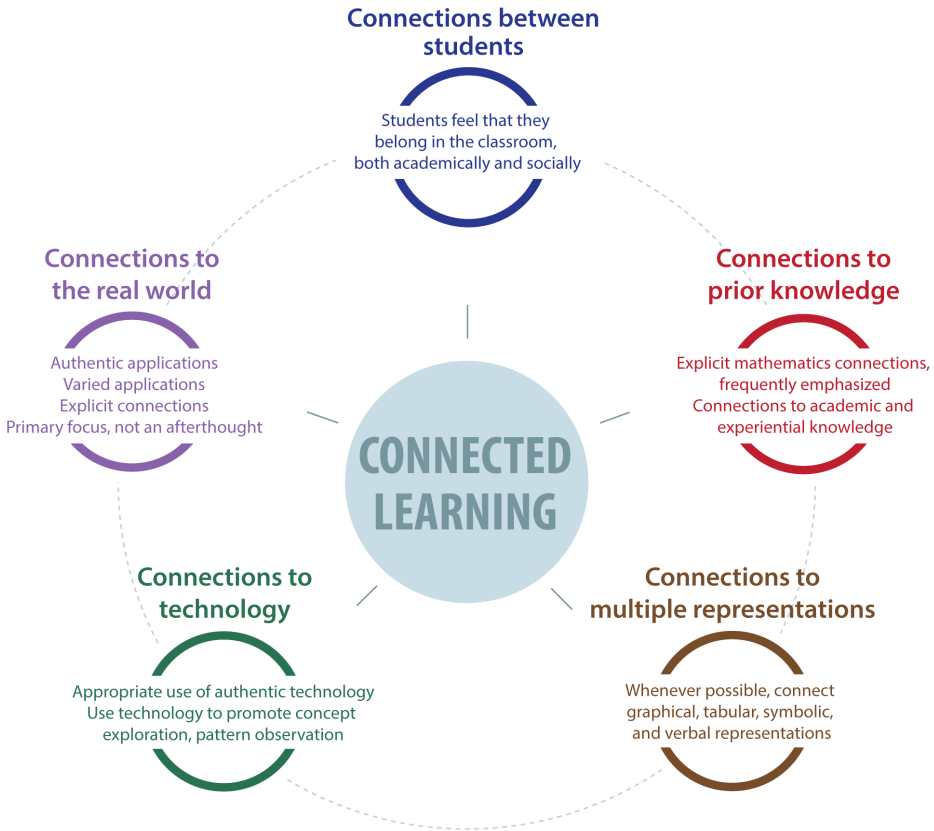
- Encouraging alternative approaches.
- Asking students about the reasonableness of their answers.
- Asking students to make explicit connections between multiple representations.
- Including new situations where student need to extend their understanding.
- Demonstrating that premises of the course are solidly based.
- Expecting students to use precise mathematical language along with understanding.
- Giving students feedback about the clarity of their reasoning.

Connected Learning

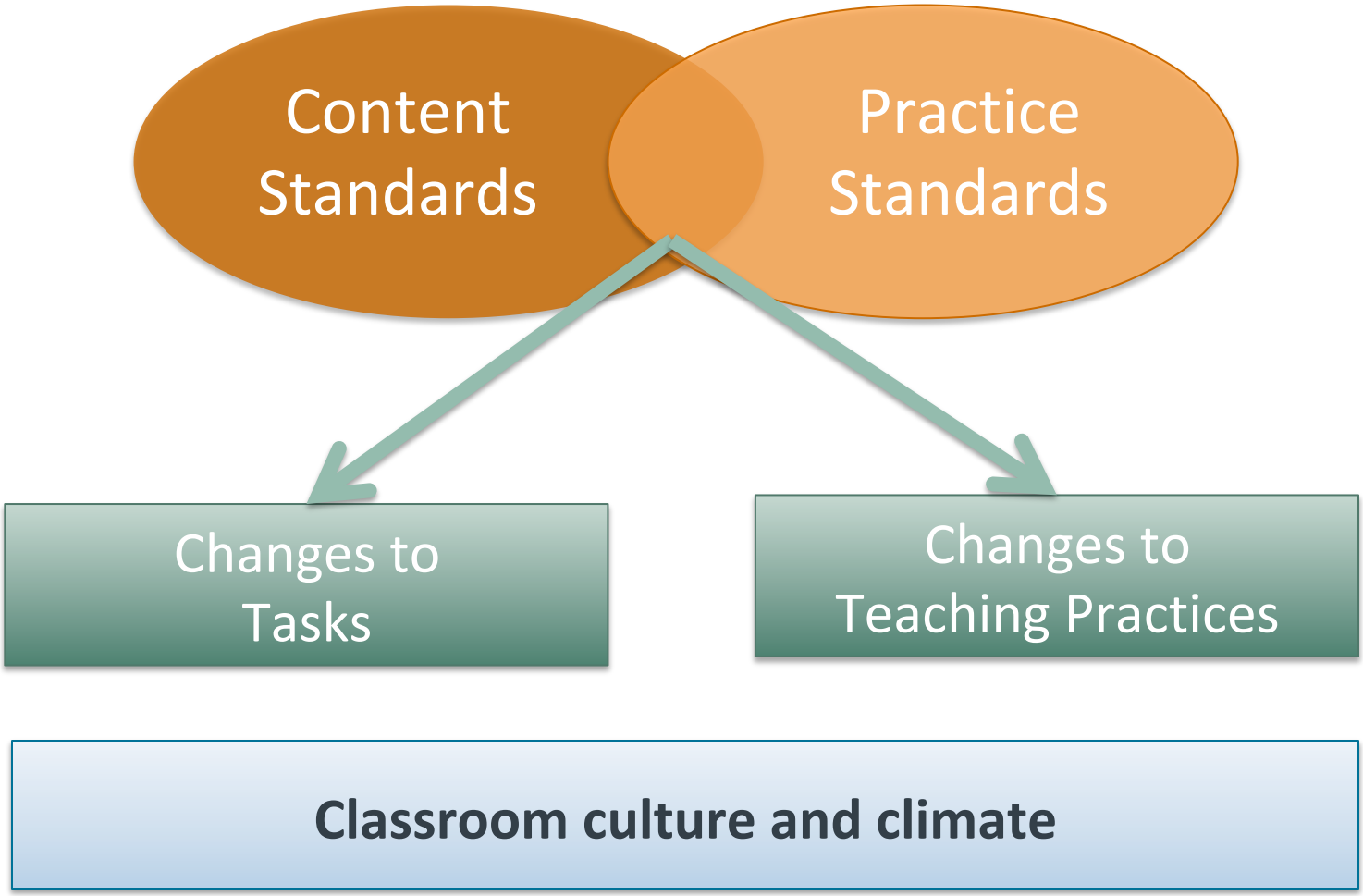


Classroom Tip ✓

Analyze your application problems. Do they provide at least three of the connections? Are the connections truly authentic and relevant?



Connecting to the K12 standards



Practice Standards

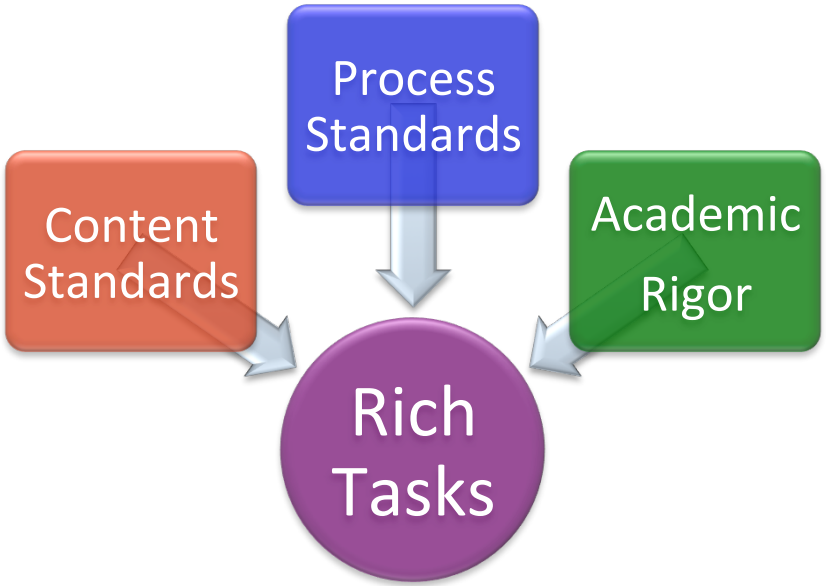
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

http://www.corestandards.org/Math/Practice/

Using Rich Tasks to Create Rigorous Learning Opportunities

Rich mathematical tasks include:

- **Students** as the workers and the decisionmakers
- High-level thinking and reasoning by **students**
- Discussion, collaboration, or active inquiry
- Multiple layers of complexity
- Multiple entry points
- Multiple solutions and/or strategies



Breakout #3: Creating Rich Tasks

Choose a reporter and a timekeeper.

In your breakout, discuss:

- *What support would you need to create and use more rich tasks in your classes?*

Share-out #3: Creating Rich Tasks

Reporter:

- *What support would you need to create and use more rich tasks in your classes?*

Other participants:

If there is anything else you want to share that has not been shared, please share in the chat box.

Planning Rigorous Content

Practice Exercises

Solve each equation in Exercises 1–14 by factoring.

1. $x^2 - 3x - 10 = 0$
2. $x^2 - 13x + 36 = 0$
3. $x^2 = 8x - 15$
4. $x^2 = -11x - 10$
5. $6x^2 + 11x - 10 = 0$
6. $9x^2 + 9x + 2 = 0$
7. $3x^2 - 2x = 8$
8. $4x^2 - 13x = -3$
9. $3x^2 + 12x = 0$
10. $5x^2 - 20x = 0$
11. $2x(x - 3) = 5x^2 - 7x$
12. $16x(x - 2) = 8x - 25$
13. $7 - 7x = (3x + 2)(x - 1)$
14. $10x - 1 = (2x + 1)^2$

Solve each equation in Exercises 15–34 by the square root property.

15. $3x^2 = 27$
16. $5x^2 = 45$
17. $5x^2 + 1 = 51$
18. $3x^2 - 1 = 47$
19. $2x^2 - 5 = -55$
20. $2x^2 - 7 = -15$
21. $(x + 2)^2 = 25$
22. $(x - 3)^2 = 36$
23. $3(x - 4)^2 = 15$
24. $3(x + 4)^2 = 21$
25. $(x + 3)^2 = -16$
26. $(x - 1)^2 = -9$
27. $(x - 3)^2 = -5$
28. $(x + 2)^2 = -7$
29. $(3x + 2)^2 = 9$
30. $(4x - 1)^2 = 16$
31. $(5x - 1)^2 = 7$
32. $(8x - 3)^2 = 5$
33. $(3x - 4)^2 = 8$
34. $(2x + 8)^2 = 27$

In Exercises 35–46, determine the constant that should be added to the binomial so that it becomes a perfect square trinomial. Then write and factor the trinomial.

35. $x^2 + 12x$
36. $x^2 + 16x$
37. $x^2 - 10x$
38. $x^2 - 14x$
39. $x^2 + 3x$
40. $x^2 + 5x$
41. $x^2 - 7x$
42. $x^2 - 9x$
43. $x^2 - \frac{2}{3}x$
44. $x^2 + \frac{4}{5}x$

In Exercises 75–82, compute the discriminant. Then determine the number and type of solutions for the given equation.

75. $x^2 - 4x - 5 = 0$
76. $4x^2 - 2x + 3 = 0$
77. $2x^2 - 11x + 3 = 0$
78. $2x^2 + 11x - 6 = 0$
79. $x^2 - 2x + 1 = 0$
80. $3x^2 = 2x - 1$
81. $x^2 - 3x - 7 = 0$
82. $3x^2 + 4x - 2 = 0$

Solve each equation in Exercises 83–108 by the method of your choice.

83. $2x^2 - x = 1$
84. $3x^2 - 4x = 4$
85. $5x^2 + 2 = 11x$
86. $5x^2 = 6 - 13x$
87. $3x^2 = 60$
88. $2x^2 = 250$
89. $x^2 - 2x = 1$
90. $2x^2 + 3x = 1$
91. $(2x + 3)(x + 4) = 1$
92. $(2x - 5)(x + 1) = 2$
93. $(3x - 4)^2 = 16$
94. $(2x + 7)^2 = 25$
95. $3x^2 - 12x + 12 = 0$
96. $9 - 6x + x^2 = 0$
97. $4x^2 - 16 = 0$
98. $3x^2 - 27 = 0$
99. $x^2 - 6x + 13 = 0$
100. $x^2 - 4x + 29 = 0$
101. $x^2 = 4x - 7$
102. $5x^2 = 2x - 3$
103. $2x^2 - 7x = 0$
104. $2x^2 + 5x = 3$
105. $\frac{1}{x} + \frac{1}{x + 2} = \frac{1}{3}$
106. $\frac{1}{x} + \frac{1}{x + 3} = \frac{1}{4}$
107. $\frac{2x}{x - 3} + \frac{6}{x + 3} = -\frac{28}{x^2 - 9}$
108. $\frac{3}{x - 3} + \frac{5}{x - 4} = \frac{x^2 - 20}{x^2 - 7x + 12}$

In Exercises 109–114, find the x-intercept(s) of the graph of each equation. Use the x-intercepts to match the equation with its graph. The graphs are shown in $[-10, 10, 1]$ *by* $[-10, 10, 1]$ *viewing rectangles and labeled (a) through (f).*

109. $y = x^2 - 4x - 5$
110. $y = x^2 - 6x + 7$

Planning Rigorous Content

The following table summarizes data from the Trust for Public Land on park area and spending for five large cities.

City	Baltimore	Chicago	Dallas	Sacramento	Seattle
Population	620,176	2,778,304	1,351,670	487,943	685,269
Park Acreage	4,917	13,547	27,038	4,959	6,591
City Area (acres)	51,318	136,796	215,676	61,972	52,765
Park Spending	$48,224,886	$477,868,288	$145,466,725	$63,647,285	$189,435,762
Park Land (acres per resident)	0.00793	0.00488	0.02000	0.01016	0.00962
Park Land (% of city area)	9.6%	9.9%	12.5%	8.0%	12.5%
Park Spending per Resident	$77.76	$172.00	$107.62	$130.44	$276.44

Based on this data, which city appears to have the most resources devoted to public parks? State your answer in complete sentences and include quantitative measures to support your conclusion.

Planning Rigorous Content

A variable is normally distributed with mean 11 and standard deviation 2.

a. Find the percentage of all possible values of the variable that lie between 7 and 16.
b. Find the percentage of all possible values of the variable that exceed 9.
c. Find the percentage of all possible values of the variable that are less than 6.

---

a. The percentage of all possible values of the variable that lie between 7 and 16 is ☐%.
(Round to two decimal places as needed.)

b. The percentage of all possible values of the variable that exceed 9 is ☐%.
(Round to two decimal places as needed.)

c. The percentage of all possible values of the variable that are less than 6 is ☐%.
(Round to two decimal places as needed.)

Increasing Rigor in Just-in-Time Supports

1. [Closely] aligning developmental course content with college-level course expectations
2. Providing consistent opportunities for students to construct knowledge [including problem solving, critical thinking, reasoning, and making predictions], and
3. Making struggle a part of the learning process

– Barragan, M., & Cormier, M. S. (2013).
Enhancing rigor in developmental education. Inside Out, 1(4)

Planning Rigorous Just-in-Time Supports

Support Course Content	College-Course Preparation Homework	College-Course Content
Operations with fractions	Convert probabilities to a “1 in ___ chance” statement	Calculate probability of independent events involving “and” and “or” statements
Chance and probability; probability notation	Determine simple and conditional probabilities of events; dependent and independent events	Calculate conditional probabilities for dependent events
Conversion factors	Dimensional analysis	Using conversions to compare data
Reference values; comparing values with percentages; reading spreadsheets	Calculate cost of living averages	Make/justify decisions and evaluate claims using index numbers
Percentages of the whole; calculating percentages with spreadsheets	Mean and weighted average	Use weighted averages to analyze data and draw conclusions
Population data and percentages; spreadsheet calculations	Sum and mean of a data set; percentages	Expected value; making predictions based on data analysis

Planning Rigorous Just-in-Time Supports

Support Course Content	College-Course Preparation Homework	College-Course Content
Arrange decimals in order; use inequalities to compare numbers; identify linear and non-linear patterns	Distinguish between linear and non-linear patterns	Use scatterplots in conjunction with their corresponding correlation coefficient values to determine the strength and type of association between two variables
Identify explanatory and response variables and types of correlations that may exist	Identify explanatory and response variables	Explain why association does not imply causation; identify potential confounding variables in situations in which a cause-and-effect conclusion is not reasonable
Use linear relationships to make predictions	Determine the -value, given the -value, using a graph or equation	Predict the value of the response variable using both the graph of a line and its equation for a scenario involving a bivariate numerical data set

Classroom Tip ✓

Provide closure each day with a Minute Paper.

- Example 1: Ask students to summarize the concept(s) for the day, using correct terminology and their own words.
  - Develop the routine that students review their Minute Papers at the start of the next class, to refresh themselves and prepare to build on that knowledge.

- Example 2: Ask students to record the questions that remain in their minds and plan an action step to get the questions answered prior to the next class.

– Angelo, T. and Cross, P.
Classroom Assessment Techniques: A Handbook for College Teachers

Breakout #4: Action Items

Choose a reporter and a timekeeper.

In your breakout, discuss:

- What are some short-term and long-term action items that you can pursue to ensure that the first year mathematics courses at your institution are rigorous?

Share-out #4: Action Items

Reporters:

- Share one short-term action item that was discussed, and one long-term action item.

Other participants:

If there are other action items that have not been shared, please share them in the chat box.

Other Resources

CSU Collaboration Spaces

- http://tiny.cc/csu-teams
- http://tiny.cc/csu-math
- http://tiny.cc/csu-english

Calendar

- www.calstate.edu/professional-development-calendar

Recordings and resources are linked to event listings in the archive.

Contact Information

- Dr. Emily Magruder, Director, CSU Institute for Teaching and Learning at emagruder@calstate.edu

  562-951-4752
- Dr. Zulmara Cline, Co-director, CSU Center for Advancement of Instruction in Quantitative Reasoning at zcline@calstate.edu

  562-951-4778
- Dr. Fred Uy, Co-director, CSU Center for Advancement of Instruction in Quantitative Reasoning at fuy@calstate.edu

  562-951-4713

Contact Information

- Connie Richardson, Manager, Higher Education Course Programs
  connie.Richardson@Austin.utexas.edu

- Joan Zoellner, Course Program Specialist
  joan.zoellner@austin.utexas.edu

- General information about the Dana Center
  www.utdanacenter.org

- DCMP Resource Site
  www.dcmathpathways.org

- To receive monthly updates about the DCMP, contact us at
  dcmathpathways@austin.utexas.edu

About the Dana Center

The **Charles A. Dana Center** at The University of Texas at Austin works with our nation's education systems to ensure that every student leaves school prepared for success in postsecondary education and the contemporary workplace.

Our work, based on research and two decades of experience, focuses on K–16 mathematics and science education with an emphasis on strategies for improving student engagement, motivation, persistence, and achievement.

We develop innovative curricula, tools, protocols, and instructional supports and deliver powerful instructional and leadership development.

2017

The University of Texas at Austin
Charles A. Dana Center

Three Key Findings on How Students Learn

- 1. Students come to the classroom with preconceptions about how the world works. If their initial understanding is not engaged, they may fail to grasp the new concepts and information that they are taught, or they may learn them for purposes of a test but revert to their preconceptions outside the classroom.

- How does this affect teaching?
  - Teachers must draw out and work with preexisting understandings that their students bring to them.

– Brown, Donovan, and Pellegrino (2000).
How People Learn: Brain, Mind, Experience, and School

Three Key Findings on How Students Learn

- 2. To develop competence in an area of inquiry, students must: (a) have a deep foundation of factual knowledge, (b) understand facts and ideas in the context of a conceptual framework, and (c) organize knowledge in ways that facilitate retrieval and application.

- How does this affect teaching?
  - Teachers must teach some subject matter in depth, providing many examples in which the same concept is at work and providing a firm foundation of factual knowledge.

Three Key Findings on How Students Learn

- 3. A “metacognitive” approach to instruction can help students learn to take control of their own learning by defining learning goals and monitoring their progress in achieving them.

- How does this affect teaching?
  - The teaching of metacognitive skills should be integrated into the curriculum in a variety of subject areas.