Session Outcomes

Participants will:

- Explore the meaning of rigor in mathematics.
- Discuss ways to promote rigor in the first year mathematics/quantitative reasoning courses.
- Engage with resources from the field and the professional associations.
Why are we exploring rigor?

These are some of the things we hear:

1) Concerns about whether it is realistic for students with weak math backgrounds to pass a rigorous college-level math course within their first year.

2) Questions about the curricular choices offered to students under math pathways (e.g. the belief that offering students statistics or quantitative reasoning, rather than a calculus-prep algebra course, is weakening the degree).

3) Speculation that offering stretch courses or support courses will lessen the rigor of the gateway math courses.
Concerns about rigor

Has the concern of rigor come up in department meetings? Enter the number of the concern in the chat box. If you choose #4, enter the reason.

1) Yes, but only as it relates to algebraic-intensive courses (Pre-Calc, College Algebra, etc.)
2) Yes, but only as it relates to non algebraic-intensive courses (Quantitative Reasoning, Statistics, etc.)
3) Yes, as it relates to offering stretch or support courses
4) Yes, but for another reason
5) No, concerns about rigor have not been raised.
Breakout #1: Sharing Thoughts About Rigor

Choose a reporter and a timekeeper.

In your breakout, discuss:

- What concerns related to rigor have come up during department discussions?
- What interests you the most about the conversation around rigor?
- What motivated you to join this webinar?
Share-out #1: Sharing Thoughts About Rigor

Reporter:

- What concerns related to rigor have come up during department discussions?
- What interests you the most about the conversation around rigor?
- What motivated you to join this webinar?

Other participants:

If there is anything else you want to share that has not been shared, please share in the chat box.
Dana Center’s Understanding of Rigor

Rigor in mathematics is a set of skills that centers on the communication and use of mathematical language.
Towards a practical view of rigor

• We should attend to all of our math courses, whether it be statistics-, modeling- or algebra-based, to ensure that they are all taught with rigor.

• To learn mathematics, all students must have the opportunity to tackle rich problems and productively struggle with them.

• They must not only solve those problems but also be able to articulate the basis of an argument at a level of precision appropriate to the course.

• Math departments should play an essential role in determining the content of their introductory courses in conjunction with the views of the professional associations and the needs of the institution’s various programs of study.
Classroom Tip

If you provide an exam study guide, mix it up! Don’t put the problems in the order of the chapter, or all of the like problems together.
Components of Rigor

- Procedural Fluency
- Communication
- Conceptual Understanding
- Application
Classroom Tip

Begin new concepts with an application problem.

Wrap-up the problem by writing the answer in a complete sentence that thoroughly answers the question.
Any Pathway Can Be Rigorous

The 17 professional associations of mathematicians which comprise the CBMS have endorsed the idea that there are many areas of mathematics that, when well taught, can serve as appropriate introductions to college mathematics and mathematical thinking and work.

http://www.cbmsweb.org/
Part II. Additional Recommendations Concerning Specific Student Audiences

A. Students taking general education or introductory college courses in the mathematical sciences

General education courses in mathematics should be designed to develop or enhance the mathematical literacy of students who will not take further mathematics courses. These courses should not be expected to provide the depth and rigor of courses that are designed for students who will take more advanced courses in mathematics. Instead, such courses should focus on developing problem-solving skills, logical reasoning, and the ability to apply mathematical concepts to real-world situations. They should also provide students with an understanding of the role of mathematics in society and its potential applications. In addition, these courses should foster an appreciation for the beauty and power of mathematics, as well as its historical and cultural significance. The content of such courses should be carefully selected to ensure that it is accessible to a wide range of students and that it can be relevant to their future academic and professional pursuits. The use of technology and interactive learning tools can be helpful in making the material more engaging and accessible.

B. Students preparing for careers in teaching mathematics

Future mathematics teachers need to develop a deep understanding of the mathematical content they will teach and the ability to communicate that content effectively. They should also have a strong foundation in the history and philosophy of mathematics, as well as an appreciation for the role of mathematics in society and its potential applications. In addition, they should be familiar with the latest research in mathematics education and the best practices for teaching mathematics at the secondary level. To ensure that they have the necessary skills to succeed in their future careers, mathematics education courses should provide students with opportunities to observe and participate in the teaching of mathematics, to engage in innovative teaching strategies, and to collaborate with experienced mathematics educators. Through these experiences, students can develop the skills and confidence they need to be effective mathematics teachers.

C. Students preparing for careers in business, engineering, and other quantitative fields

The mathematics courses that are designed for students preparing for careers in these fields should focus on developing the analytical and problem-solving skills that are necessary for success in these areas. These courses should also provide students with a strong foundation in the mathematical concepts and techniques that are used in these fields. In addition, they should be designed to help students understand the role of mathematics in these fields and how it is used to solve real-world problems. Through these courses, students can develop the skills and knowledge they need to succeed in their future careers.

D. Students preparing for careers in health sciences

The mathematics courses that are designed for students preparing for careers in the health sciences should focus on developing the analytical and problem-solving skills that are necessary for success in these fields. These courses should also provide students with a strong foundation in the mathematical concepts and techniques that are used in these fields. In addition, they should be designed to help students understand the role of mathematics in the health sciences and how it is used to solve real-world problems. Through these courses, students can develop the skills and knowledge they need to succeed in their future careers.

E. Students preparing for careers in the social sciences

The mathematics courses that are designed for students preparing for careers in the social sciences should focus on developing the analytical and problem-solving skills that are necessary for success in these fields. These courses should also provide students with a strong foundation in the mathematical concepts and techniques that are used in these fields. In addition, they should be designed to help students understand the role of mathematics in the social sciences and how it is used to solve real-world problems. Through these courses, students can develop the skills and knowledge they need to succeed in their future careers.

F. Students preparing for careers in the arts

The mathematics courses that are designed for students preparing for careers in the arts should focus on developing the analytical and problem-solving skills that are necessary for success in these fields. These courses should also provide students with a strong foundation in the mathematical concepts and techniques that are used in these fields. In addition, they should be designed to help students understand the role of mathematics in the arts and how it is used to solve real-world problems. Through these courses, students can develop the skills and knowledge they need to succeed in their future careers.
Breakout #2: MAA CUPM Article

Choose a reporter and a timekeeper.

In your breakout, discuss:

- What information did you find that resonates with you?
Share-out #2: MAA CUPM Article

**Reporter:**

- *What information did you find that resonates with you?*

**Other participants:**

If there is anything else you want to share that has not been shared, please share in the chat box.
Is the following a characteristic of a rigorous course?

Provide connections among mathematical ideas

Yes  No
Is the following a characteristic of a rigorous course?

Require memorization of rules and procedures and use of a rote procedure to solve problems

Yes  No
Is the following a characteristic of a rigorous course?

Teachers doing the work while students watch

Yes  No
Is the following a characteristic of a rigorous course?

Students know how to perform a list of algebraic tasks such as: multi-step factoring, rationalizing $n^{th}$ roots, completing the square, etc.

Yes  No
# Learning Experiences

<table>
<thead>
<tr>
<th>Learning experiences that involve rigor ...</th>
<th>Experiences that do not involve rigor ...</th>
</tr>
</thead>
<tbody>
<tr>
<td>challenge students</td>
<td>are more “difficult,” with no purpose (overly-complicated polynomial long division)</td>
</tr>
<tr>
<td>require effort and tenacity by students</td>
<td>require minimal effort</td>
</tr>
<tr>
<td>focus on quality (rich tasks)</td>
<td>focus on quantity (more pages to do)</td>
</tr>
<tr>
<td>include entry points and extensions for all students</td>
<td>are offered only to gifted students</td>
</tr>
</tbody>
</table>

[https://www.nctm.org/News-and-Calendar/Messages-from-the-President/Archive/Linda-M_Gojak/What_s-All-This-Talk-about-Rigor/](https://www.nctm.org/News-and-Calendar/Messages-from-the-President/Archive/Linda-M_Gojak/What_s-All-This-Talk-about-Rigor/)
<table>
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<tr>
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<th>Experiences that do not involve rigor ...</th>
</tr>
</thead>
<tbody>
<tr>
<td>provide connections among mathematical ideas</td>
<td>do not connect to other mathematical ideas</td>
</tr>
<tr>
<td>contain rich mathematics that is relevant to students</td>
<td>contain routine procedures with little relevance</td>
</tr>
<tr>
<td>develop strategic and flexible thinking</td>
<td>follow a rote procedure</td>
</tr>
<tr>
<td>encourage reasoning and sense making</td>
<td>require memorization of rules and procedures without understanding</td>
</tr>
<tr>
<td>expect students to be actively involved in their own learning</td>
<td>often involve teachers doing the work while students watch</td>
</tr>
</tbody>
</table>

[https://www.nctm.org/News-and-Calendar/Messages-from-the-President/Archive/Linda-M_Gojak/What_s-All-This-Talk-about-Rigor_/](https://www.nctm.org/News-and-Calendar/Messages-from-the-President/Archive/Linda-M_Gojak/What_s-All-This-Talk-about-Rigor_/)
Challenges with ensuring rigor

What are the biggest challenges to ensuring that your gateway math courses are rigorous?

Enter the number of the challenge in the chat box. If you choose #5, enter the challenge in the chat.

1) Making explicit connections between concepts
2) Using relevant mathematical scenarios
3) Helping students develop strategies that make sense to them, rather than relying on memorization of rote procedures
4) Encouraging students to work actively and take control of their learning
5) Other
Activities and Assignments That Promote Rigor:

- Encouraging alternative approaches.
- Asking students about the reasonableness of their answers.
- Asking students to make explicit connections between multiple representations.
- Including new situations where students need to extend their understanding.
- Demonstrating that premises of the course are solidly based.
- Expecting students to use precise mathematical language along with understanding.
- Giving students feedback about the clarity of their reasoning.
Connected Learning

Connections between students
- Students feel that they belong in the classroom, both academically and socially

Connections to the real world
- Authentic applications
- Varied applications
- Explicit connections
- Primary focus, not an afterthought

Connections to prior knowledge
- Explicit mathematics connections, frequently emphasized
- Connections to academic and experiential knowledge

Connections to technology
- Appropriate use of authentic technology
- Use technology to promote concept exploration, pattern observation

Connections to multiple representations
- Whenever possible, connect graphical, tabular, symbolic, and verbal representations
Classroom Tip

Analyze your application problems. Do they provide at least three of the connections? Are the connections truly authentic and relevant?
Connecting to the K12 standards

Content Standards

Practice Standards

Changes to Tasks

Changes to Teaching Practices

Classroom culture and climate
Practice Standards

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

http://www.corestandards.org/Math/Practice/
Using Rich Tasks to Create Rigorous Learning Opportunities

Rich mathematical tasks include:

• **Students** as the workers and the decisionmakers
• High-level thinking and reasoning by **students**
• Discussion, collaboration, or active inquiry
• Multiple layers of complexity
• Multiple entry points
• Multiple solutions and/or strategies

Rich Tasks
- Process Standards
- Content Standards
- Academic Rigor

Dana Center
Mathematics PATHWAYS
Breakout #3: Creating Rich Tasks

Choose a reporter and a timekeeper.

In your breakout, discuss:

- What support would you need to create and use more rich tasks in your classes?
Share-out #3: Creating Rich Tasks

Reporter:

- What support would you need to create and use more rich tasks in your classes?

Other participants:

If there is anything else you want to share that has not been shared, please share in the chat box.
Planning Rigorous Content

Practice Exercises

Solve each equation in Exercises 1–14 by factoring.

1. \( x^2 - 3x - 10 = 0 \)
2. \( x^2 = 13x + 36 = 0 \)
3. \( x^2 = 8x - 15 \)
4. \( x^2 = -11x - 10 \)
5. \( 6x^2 + 11x + 10 = 0 \)
6. \( 9x^2 + 9x + 2 = 0 \)
7. \( 3x^2 - 2x = 8 \)
8. \( 4x^2 - 13x = -3 \)
9. \( 3x^2 + 12x = 0 \)
10. \( 5x^2 - 20x = 0 \)
11. \( 2x(x - 3) = 5x^2 - 7x \)
12. \( 16x(x - 2) = 8x - 25 \)
13. \( 7 - 7x = (3x + 2)(x - 1) \)
14. \( 10x - 1 = (2x + 1)^2 \)

Solve each equation in Exercises 15–34 by the square root property.

15. \( 3x^2 = 27 \)
16. \( 5x^2 = 45 \)
17. \( 5x^2 + 1 = 51 \)
18. \( 3x^2 - 1 = 47 \)
19. \( 2x^2 - 5 = -55 \)
20. \( 2x^2 - 7 = -15 \)
21. \( (x + 2)^2 = 25 \)
22. \( (x - 3)^2 = 36 \)
23. \( 3(x - 4)^2 = 15 \)
24. \( 3(x + 4)^2 = 21 \)
25. \( (x + 3)^2 = -16 \)
26. \( (x - 1)^2 = -9 \)
27. \( (x - 3)^2 = -5 \)
28. \( (x + 2)^2 = -7 \)
29. \( (3x + 2)^2 = 9 \)
30. \( (4x - 1)^2 = 16 \)
31. \( (5x - 1)^2 = 7 \)
32. \( (8x - 3)^2 = 5 \)
33. \( (3x - 4)^2 = 8 \)

In Exercises 35–46, determine the constant that should be added to the binomial so that it becomes a perfect square trinomial. Then write and factor the trinomial.

35. \( x^2 + 12x \)
36. \( x^2 + 16x \)
37. \( x^2 - 10x \)
38. \( x^2 = 14x \)
39. \( x^2 + 3x \)
40. \( x^2 + 5x \)
41. \( x^2 - 7x \)
42. \( x^2 - 9x \)
43. \( x^2 = \frac{2}{3} \)
44. \( x^2 + \frac{2}{5} \)

In Exercises 75–82, compute the discriminant. Then determine the number and type of solutions for the given equation.

75. \( x^2 - 4x - 5 = 0 \)
76. \( 4x^2 - 2x + 3 = 0 \)
77. \( 2x^2 - 11x + 3 = 0 \)
78. \( 2x^2 + 11x - 6 = 0 \)
79. \( x^2 - 2x + 1 = 0 \)
80. \( 3x^2 = 2x - 1 \)
81. \( x^2 - 3x - 7 = 0 \)
82. \( 3x^2 + 4x - 2 = 0 \)

Solve each equation in Exercises 83–108 by the method of your choice.

83. \( 2x^2 - x = 1 \)
84. \( 3x^2 - 4x = 4 \)
85. \( 5x^2 + 2 = 11x \)
86. \( 5x^2 = 6 - 13x \)
87. \( 3x^2 = 60 \)
88. \( 2x^2 = 250 \)
89. \( x^2 - 2x = 1 \)
90. \( 2x^2 + 3x = 1 \)
91. \( (2x + 3)(x + 4) = 1 \)
92. \( (2x - 5)(x + 1) = 2 \)
93. \( (3x - 4)^2 = 16 \)
94. \( (2x + 7)^2 = 25 \)
95. \( 3x^2 = 12x + 12 = 0 \)
96. \( 9 - 6x + x^2 = 0 \)
97. \( 4x^2 - 16 = 0 \)
98. \( 3x^2 = 27 = 0 \)
99. \( x^2 - 6x + 13 = 0 \)
100. \( x^2 = 4x + 29 = 0 \)
101. \( x^2 - 4x = 7 \)
102. \( 5x^2 = 2x - 3 \)
103. \( 2x^2 - 7x = 0 \)
104. \( 2x^2 + 5x = 3 \)
105. \( \frac{1}{x} + \frac{1}{x + 2} = \frac{1}{3} \)
106. \( \frac{1}{x} + \frac{1}{x + 3} = \frac{1}{4} \)
107. \( \frac{2x}{x - 3} + \frac{6}{x + 3} = -\frac{28}{x^2 - 9} \)
108. \( \frac{3}{x - 3} + \frac{5}{x - 4} = \frac{x^2 - 20}{x^2 - 7x + 12} \)

In Exercises 109–114, find the x-intercept(s) of the graph of each equation. Use the x-intercepts to match the equation with its graph. The graphs are shown in \([-10, 10, 1]\) by \([-10, 10, 1]\) viewing rectangles and labeled (a) through (f).

109. \( y = x^2 - 4x - 5 \)
110. \( y = x^2 - 6x + 7 \)
Planning Rigorous Content

The following table summarizes data from the Trust for Public Land on park area and spending for five large cities.

<table>
<thead>
<tr>
<th>City</th>
<th>Baltimore</th>
<th>Chicago</th>
<th>Dallas</th>
<th>Sacramento</th>
<th>Seattle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population</td>
<td>620,176</td>
<td>2,778,304</td>
<td>1,351,670</td>
<td>487,943</td>
<td>685,269</td>
</tr>
<tr>
<td>Park Acreage</td>
<td>4,917</td>
<td>13,547</td>
<td>27,038</td>
<td>4,959</td>
<td>6,591</td>
</tr>
<tr>
<td>City Area (acres)</td>
<td>51,318</td>
<td>136,796</td>
<td>215,676</td>
<td>61,972</td>
<td>52,765</td>
</tr>
<tr>
<td>Park Spending</td>
<td>$48,224,886</td>
<td>$477,868,288</td>
<td>$145,466,725</td>
<td>$63,647,285</td>
<td>$189,435,762</td>
</tr>
<tr>
<td>Park Land (acres per resident)</td>
<td>0.00793</td>
<td>0.00488</td>
<td>0.02000</td>
<td>0.01016</td>
<td>0.00962</td>
</tr>
<tr>
<td>Park Land (% of city area)</td>
<td>9.6%</td>
<td>9.9%</td>
<td>12.5%</td>
<td>8.0%</td>
<td>12.5%</td>
</tr>
<tr>
<td>Park Spending per Resident</td>
<td>$77.76</td>
<td>$172.00</td>
<td>$107.62</td>
<td>$130.44</td>
<td>$276.44</td>
</tr>
</tbody>
</table>

Based on this data, which city appears to have the most resources devoted to public parks? State your answer in complete sentences and include quantitative measures to support your conclusion.
Planning Rigorous Content

A variable is normally distributed with mean 11 and standard deviation 2.

a. Find the percentage of all possible values of the variable that lie between 7 and 16.
b. Find the percentage of all possible values of the variable that exceed 9.
c. Find the percentage of all possible values of the variable that are less than 6.

a. The percentage of all possible values of the variable that lie between 7 and 16 is \[ \square \]%.
   (Round to two decimal places as needed.)

b. The percentage of all possible values of the variable that exceed 9 is \[ \square \]%.
   (Round to two decimal places as needed.)

c. The percentage of all possible values of the variable that are less than 6 is \[ \square \]%.
   (Round to two decimal places as needed.)
Increasing Rigor in Just-in-Time Supports

1. [Closely] aligning developmental course content with college-level course expectations
2. Providing consistent opportunities for students to construct knowledge [including problem solving, critical thinking, reasoning, and making predictions], and
3. Making struggle a part of the learning process

### Planning Rigorous Just-in-Time Supports

<table>
<thead>
<tr>
<th>Support Course Content</th>
<th>College-Course Preparation Homework</th>
<th>College-Course Content</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operations with fractions</td>
<td>Convert probabilities to a “1 in ___ chance” statement</td>
<td>Calculate probability of independent events involving “and” and “or” statements</td>
</tr>
<tr>
<td>Chance and probability; probability notation</td>
<td>Determine simple and conditional probabilities of events; dependent and independent events</td>
<td>Calculate conditional probabilities for dependent events</td>
</tr>
<tr>
<td>Conversion factors</td>
<td>Dimensional analysis</td>
<td>Using conversions to compare data</td>
</tr>
<tr>
<td>Reference values; comparing values with percentages; reading spreadsheets</td>
<td>Calculate cost of living averages</td>
<td>Make/justify decisions and evaluate claims using index numbers</td>
</tr>
<tr>
<td>Percentages of the whole; calculating percentages with spreadsheets</td>
<td>Mean and weighted average</td>
<td>Use weighted averages to analyze data and draw conclusions</td>
</tr>
<tr>
<td>Population data and percentages; spreadsheet calculations</td>
<td>Sum and mean of a data set; percentages</td>
<td>Expected value; making predictions based on data analysis</td>
</tr>
</tbody>
</table>
## Planning Rigorous Just-in-Time Supports

<table>
<thead>
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</thead>
<tbody>
<tr>
<td>Arrange decimals in order; use inequalities to compare numbers; identify linear and non-linear patterns</td>
<td>Distinguish between linear and non-linear patterns</td>
<td>Use scatterplots in conjunction with their corresponding correlation coefficient values to determine the strength and type of association between two variables</td>
</tr>
<tr>
<td>Identify explanatory and response variables and types of correlations that may exist</td>
<td>Identify explanatory and response variables</td>
<td>Explain why association does not imply causation; identify potential confounding variables in situations in which a cause-and-effect conclusion is not reasonable</td>
</tr>
<tr>
<td>Use linear relationships to make predictions</td>
<td>Determine the -value, given the -value, using a graph or equation</td>
<td>Predict the value of the response variable using both the graph of a line and its equation for a scenario involving a bivariate numerical data set</td>
</tr>
</tbody>
</table>
Classroom Tip

Provide closure each day with a Minute Paper.

- Example 1: Ask students to summarize the concept(s) for the day, using correct terminology and their own words.
  - Develop the routine that students review their Minute Papers at the start of the next class, to refresh themselves and prepare to build on that knowledge.

- Example 2: Ask students to record the questions that remain in their minds and plan an action step to get the questions answered prior to the next class.

– Angelo, T. and Cross, P.

*Classroom Assessment Techniques: A Handbook for College Teachers*
Breakout #4: Action Items

Choose a reporter and a timekeeper.

In your breakout, discuss:

• What are some short-term and long-term action items that you can pursue to ensure that the first year mathematics courses at your institution are rigorous?
Share-out #4: Action Items

Reporters:
• Share one short-term action item that was discussed, and one long-term action item.

Other participants:
If there are other action items that have not been shared, please share them in the chat box.
Other Resources

CSU Collaboration Spaces

- [http://tiny.cc/csu-teams](http://tiny.cc/csu-teams)
- [http://tiny.cc/csu-math](http://tiny.cc/csu-math)
- [http://tiny.cc/csu-english](http://tiny.cc/csu-english)

Calendar

- [www.calstate.edu/professional-development-calendar](http://www.calstate.edu/professional-development-calendar)

Recordings and resources are linked to event listings in the archive.
Contact Information

• Dr. Emily Magruder, Director, CSU Institute for Teaching and Learning at emagruder@calstate.edu
  562-951-4752

• Dr. Zulmara Cline, Co-director, CSU Center for Advancement of Instruction in Quantitative Reasoning at zcline@calstate.edu
  562-951-4778

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• General information about the Dana Center
  www.utdanacenter.org

• DCMP Resource Site
  www.dcmathpathways.org

• To receive monthly updates about the DCMP, contact us at
  dcmathpathways@austin.utexas.edu
The Charles A. Dana Center at The University of Texas at Austin works with our nation’s education systems to ensure that every student leaves school prepared for success in postsecondary education and the contemporary workplace.

Our work, based on research and two decades of experience, focuses on K–16 mathematics and science education with an emphasis on strategies for improving student engagement, motivation, persistence, and achievement.

We develop innovative curricula, tools, protocols, and instructional supports and deliver powerful instructional and leadership development.
Three Key Findings on How Students Learn

1. Students come to the classroom with preconceptions about how the world works. If their initial understanding is not engaged, they may fail to grasp the new concepts and information that they are taught, or they may learn them for purposes of a test but revert to their preconceptions outside the classroom.

- How does this affect teaching?
  - Teachers must draw out and work with preexisting understandings that their students bring to them.

Three Key Findings on How Students Learn

- 2. To develop competence in an area of inquiry, students must: (a) have a deep foundation of factual knowledge, (b) understand facts and ideas in the context of a conceptual framework, and (c) organize knowledge in ways that facilitate retrieval and application.

- How does this affect teaching?
  - Teachers must teach some subject matter in depth, providing many examples in which the same concept is at work and providing a firm foundation of factual knowledge.
Three Key Findings on How Students Learn

- 3. A “metacognitive” approach to instruction can help students learn to take control of their own learning by defining learning goals and monitoring their progress in achieving them.

- How does this affect teaching?
  - The teaching of metacognitive skills should be integrated into the curriculum in a variety of subject areas.