# Getting Students to Think About Math By Stopping Them From Calculating

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# This Webinar

1. Exploring Students’ Approach to Math
   - One-On-One Interviews (CC)
   - Mock Class (CSU)

2. Changing Students’ Approach to Math
   - Intervention: Removing Opportunities to Calculate (CC)

# CC Interviews

- N = 30
- Arithmetic, Pre-Algebra, and Elementary Algebra
- Mean = 75 minutes

# Lack a disposition to think when doing math

"What does it mean to be good at math?"

- 77% gave answers like these:
  - "Math is just all these steps."
  - "In math, sometimes you have to just accept that that's the way it is and there's no reason behind it."
  - "I don't think [being good at math] has anything to do with reasoning. It's all memorization."

# Maladaptive views on how to get better at math

“What advice would you give to your instructor about how to better help you learn math?”

- Dominant themes:
  - Present material more slowly and with more repetition
  - Break down procedures into smaller steps

# Math as a Collection of Procedures to be Applied

- We taught them that procedures are important.
- Students’ actions reflect their conceptions.

# Math as a Collection of Procedures to be Applied

"Place the following fractions on a number line:"

$$\frac{4}{5}, \frac{5}{8}, -\frac{3}{4}, \frac{5}{4}$$

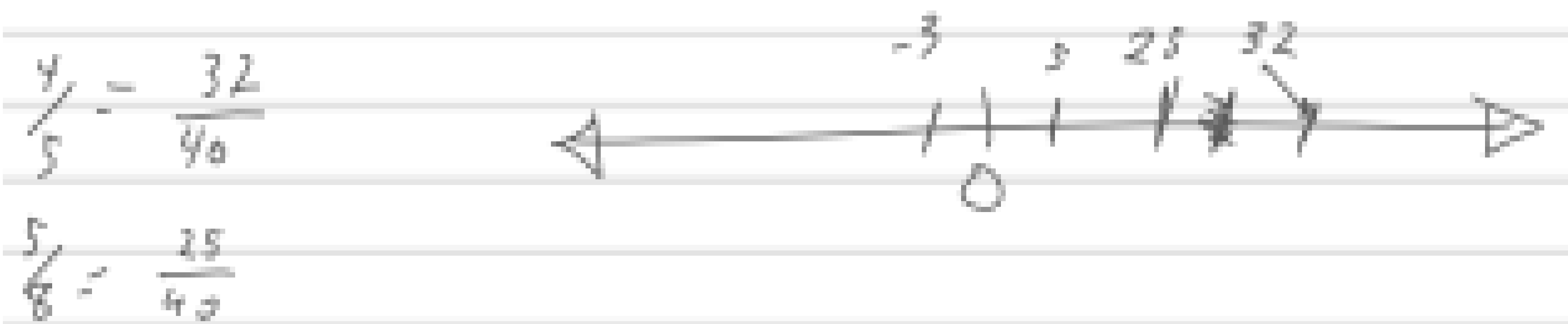
- 30% of students set about dividing
- 2/3 of those who used division obtained a new fraction not equivalent to the first

$$\frac{4}{5} \rightarrow 1\frac{1}{4} \rightarrow \frac{5}{4}$$

# Math as a Collection of Procedures to be Applied

Place these numbers on a number line: $\frac{4}{5}, \frac{5}{8}$

Now add these: $-\frac{3}{4}, \frac{5}{4}$



# Math as a Collection of Procedures to be Applied

Select the larger value, given

$$\frac{a}{5} \qquad \frac{a}{8}$$

When prompted to substitute for *a*, one student:

$$\frac{1}{5} \times \frac{1}{8} = \frac{1}{40}$$

# Math as a Collection of Procedures to be Applied

What would happen if you had a number and you added 1/3 to it? Would it be more than what you started with, less than what you started with, the same as what you started with, or can you not tell?

- 87% answered correctly

- If $a+\frac{1}{3}=x$, is *x* more than *a*, less than *a*, the same as *a*, or can you not tell?
  - 30% said unanswerable, unless *a* or *x* is known
  - 78% of those had just answered the first question correctly

# Poll

Have you been guilty of leading students to think that math is a series of procedures?

- Yes
- No

# Why Does It Matter?

Please type in the Open Chat

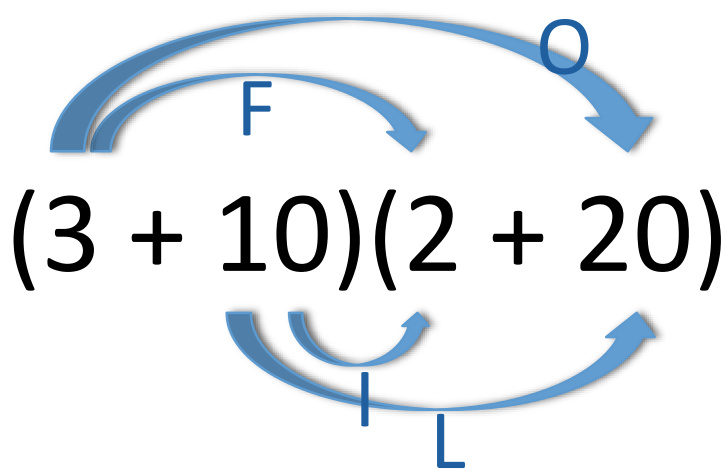
# Procedures Memorized without Meaning

When we use the standard algorithm to solve 22 x 13,
Why do we put a “0” here?

$$
\begin{array}{r}
22 \\
\underline{\times\ 13} \\
66 \\
\underline{220} \\
286
\end{array}
$$

$$
\begin{array}{r l}
22 & \\
\underline{\times\ 13} & \\
66 & \leftarrow 3 \times 22 \\
\underline{220} & \leftarrow 10 \times 22 \\
286 &
\end{array}
$$

$$
\begin{array}{r l}
22 & \\
\underline{\times\ 13} & \\
6 & \leftarrow 3 \times 2 \\
60 & \leftarrow 3 \times 20 \\
20 & \leftarrow 10 \times 2 \\
\underline{200} & \leftarrow 10 \times 20 \\
286 &
\end{array}
$$



When we use the standard algorithm to solve 22 x 13,
Why do we put a “0” here?

$$\begin{array}{r} 22 \\ \underline{\times\ 13} \\ 66 \\ \underline{220} \\ 286 \end{array}$$

# Interview Take-Aways

Students learn the math we teach them.

- Math isn't something to *think* about.
- Math is a collection of procedures to be applied.
- Procedures should be memorized (without regard to meaning).

# A Final Example

$$\begin{array}{r} 462 \\ +\quad 253 \\ \hline 715 \end{array}$$

"Check your answer using subtraction."

Student subtracted 253 from 715.

video

# “The people who created math”

California State University student

Discussing a problem involving the division of fractions.

video

# “What is half of two-thirds”

California State University student

Discussing a problem involving the division of fractions.

# Skemp 1976

**Instrumental understanding**:

rules without reasons

**Relational understanding**:

knowing what to do and why

How can we get students to think relationally rather than instrumentally about math (in a way that's scalable)?

*Stop them from executing procedures.*

How can we stop them from executing procedures (in a way that's scalable)?

*Remove opportunities to calculate from the problems we give them.*

# Two Studies

- Colleges in LACCD
- Introductory and Elementary Algebra
- 30 minutes of class time

# What was atypical about what we did?

- The Problems in the Intervention
  - ½ of students received problems that included values with which they could calculate (*typical*)
  - ½ of students received problems without values with which they could calculate
- The Instructions in the Intervention
  - Explain the problem to another student

# In 30 minutes of seatwork…

- Intervention: 3 “explain” problems
- Outcome Measure: 4 “solve” problems
  - 3 similar to intervention, 1 transfer

# Sample Intervention Problems

- Including values with which to calculate

  Andrew was planning a party for 20 people. He found a bakery that had amazing cupcakes, and wanted to make sure each person could have at least 2. The bakery only sells the cupcakes in boxes of 6. How many boxes does he need to buy?

- Opportunities to calculate removed

  Andrew was planning a big party for his friends. He found a bakery that had amazing cupcakes, and wanted to make sure each person could have seconds. The bakery only sells the cupcakes in boxes of six. How many boxes does he need to buy?

# In the Chat Pod…

- If you were a student, how would you start your explanation of the second problem?

# Anxiety Scale

On a scale from 1 to 10, how math anxious are you?

1 (not anxious) to 10 (very anxious)

# Results – Outcome Measure

- *Across all 4 items*: Students in the no numbers condition outscored students in the numbers condition.

  (mean = 2.00 vs. 1.25)

- *Transfer item*: More participants in the no numbers group got the single, transfer item correct than did participants in the numbers group.

  (10 of 41 vs. 3 of 40)

# Students' Explanations

- Participants in the numbers group more frequently SOLVED the "explain" problems than participants in the no numbers group.
- Students in the no number group less frequently described steps than did students in the number group.

# A Need for Numbers

Students in the no number group:

- 29% stated explicitly in at least one of their responses that numbers were needed in order to answer
- 44% at least once made up numbers in order to solve and respond.

# Math Anxiety

- Participants reported moderate levels of math anxiety, with levels similar across conditions.
  (*M* = 6.45 out of 10)
- Anxiety was negatively associated with performance on the outcome measures, within both groups.
- Within the no numbers group, anxiety was significantly correlated with the frequency with which students made up values in order solve the problems.

# Conclusion

- We didn't suppress students' drive to calculate
- Students struggled to provide explanations
  - Lacked relational understanding?
  - With little experience, they have a poor grasp of what it means to explain?
  - Assume experimenter <u>must</u> have wanted calculations?

- Maybe struggling to understand what was being requested was sufficient
- Maybe simply slowing down students’ ability to calculate affords new opportunities for thinking

# Remaining Questions for Future Research

1. Does the effect generalize to different kinds of math problems, including more complex ones?
2. How else can relational thinking be supported?
3. How might the effect be strengthened by more extended opportunities for sense-making?
4. Does the effect last?

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