

# Sample Preparatory Assignment

## Working with Functions

In the next class, you will need to be able to calculate the percent change between two values, write a formula for an **exponential function** given its starting value and percent rate of change, relate the base of an exponential function to its percent rate of change, determine whether an exponential function is increasing or decreasing, and use interval notation. You will also need to analyze **periodic functions**, identify the period of each function, and identify when the functions are increasing and when they are decreasing. Don't worry if you are unfamiliar with some of these terms; they will be defined below as needed.

### Percent Change Between Two Values

When a quantity changes from an initial value,  $P_1$ , to a new value  $P_2$ , the **percent change** may be calculated by dividing the change in the values by the initial value. Sometimes this is also called the **relative change**. In symbols,

$$\frac{P_2 - P_1}{P_1}.$$

Be aware that the change in values (the top part of the fraction) is computed by subtracting the initial value from the new value.

For example, if a population grows from 800 to 863, then the percent change in the population is

$$\frac{863 - 800}{800} = 0.07875 \approx 7.9\%.$$

1) Calculate the percent change in these two situations.

Part A: The population of a town grows from 2500 to 2580.

Part B: The number of goats on an island declines from 250 to 210.

### Exponential Growth and Decay

A function such as  $f(x) = 10(1.12)^x$  is an example of an **exponential function**. In this example, the number 1.12 is called the **base**. Notice that in an exponential function, the independent variable appears as the **exponent**. You may have seen exponential functions when working with compound interest. They arise in many other situations as well. You will work with exponential functions on the next page.

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- 2) Tim hears that guppies are relatively easy to breed because they reproduce quickly.<sup>1</sup> A typical population of guppies may grow at a rate of 26% per month. Tim decides to breed guppies, so he goes to a pet store and buys 40 guppies.

Part A: Approximately how many guppies will he have after one month? After two months? After three months? Complete the table.

Months Elapsed	Calculation	Number of Guppies
1	$40 + 40 \times 0.26 = 40(1 + 0.26) = 40(1.26)$	50 (round down from 50.4)
2		
3		

Part B: Can you determine the approximate number of guppies Tim will have after six months without calculating the numbers for four and five months? Explain.

Part C: Using patterns discovered in answering the question in Part B, write an exponential function  $g$  modeling the number of guppies Tim will have after  $x$  months, assuming he begins with 40 guppies and the population grows at a rate of 26% per month. Be sure to use correct function notation.

Part D: What is the base of your exponential function? How is it related to the growth rate of 26%?

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<sup>1</sup> Information on guppy reproduction retrieved from <http://animals.pawnation.com/fast-guppies-multiply-4229.html> on November 1, 2014.

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3) Use the exponential function  $f(x) = 4000(0.8)^x$  to answer the following questions.

Part A: Use your calculator to complete the table.

$x$	$f(x)$
0	
1	
2	
3	
4	

Part B: Is this function increasing or decreasing?

Part C: What is the percent rate of change?

Part D: How is the percent rate of change related to the base of the exponential function?

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## Interval Notation

In this course, we will speak often about different segments of the real number line. For example, we might talk about all values of the variable  $x$  between 2.5 and 3.2. You may have seen this written as  $2.5 < x < 3.2$ . If we wanted to include the value 2.5 as a possible value of  $x$ , we would write  $2.5 \leq x < 3.2$ . **Interval notation** gives us a good way to express intervals like these.

Some examples are shown in the table:

Inequalities	Interval Notation
$-3 \leq x \leq 2$	$[-3, 2]$
$-3 < x \leq 2$	$(-3, 2]$
$-3 < x < 2$	$(-3, 2)$
$-3 \leq x < 2$	$[-3, 2)$

When using interval notation,

1. List the two endpoints, with the smaller one first.
2. If an endpoint is included in the set, brackets, [ or ], are used; if the endpoint is not included, parentheses, ( or ), are used instead.

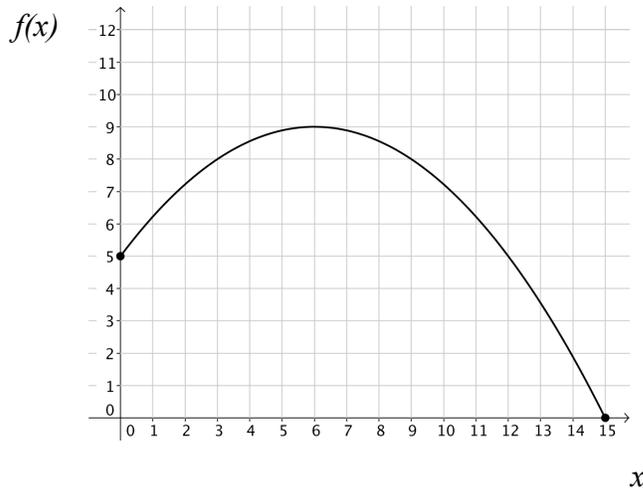
4) Write, using interval notation:

Part A:  $-11 < x \leq -2$

Part B: "All numbers between 4 and 7, not including the endpoints."

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- 5) The graph of a function  $f$  is given below. Use the graph to decide whether each of the statements is true or false. If you decide it is False, try to rewrite the statement so that it is true.



Part A:  $f(x) > 8$  when  $x$  is chosen from the interval  $(3, 9)$ .

Part B: The function is decreasing on the interval  $(0, 6)$ .

Part C:  $f(x) < 5$  when  $x$  is chosen from the interval  $(12, 15)$ .

Part D: The function is decreasing on the interval  $(6, 15)$ .

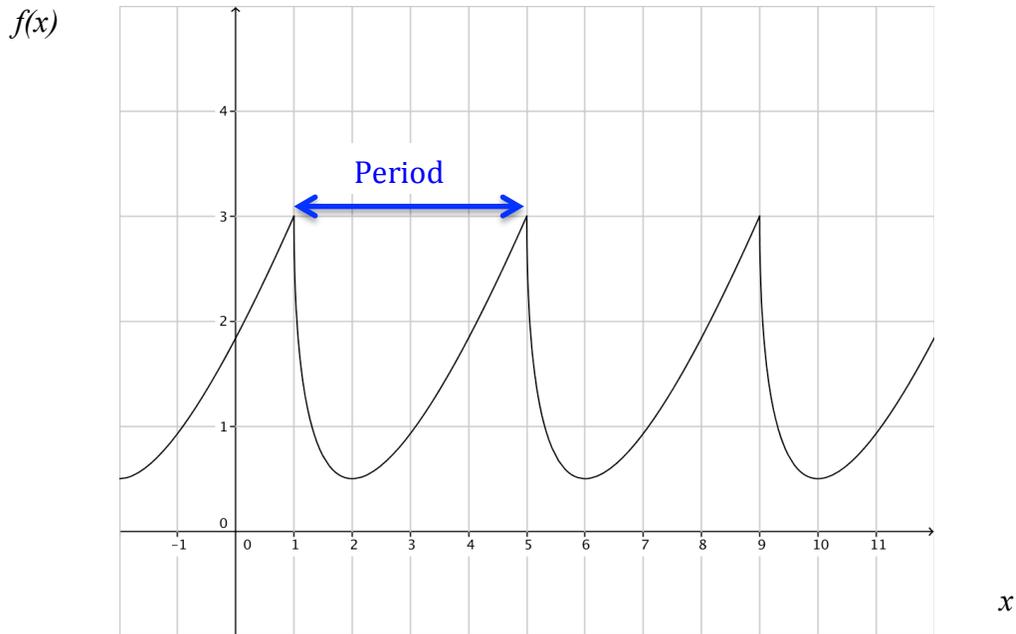
- 6) Write down one or two questions you want to ask in class in order to better understand how to use interval notation.

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## Periodic Functions

A function that repeats its values over regular intervals is called a **periodic function**. The length of that regular interval is called the **period** of the function.

7) Use the graph of the periodic function shown below to answer the questions.



Part A: What is the value of  $f(1)$ ?

Part B: According to the graph, what other values of  $x$  have the same function value as  $f(1)$ ? (Hint: You should find two more  $x$  values that have the same  $y$ -value as Part A.)

Part C: All of the  $y$ -values in Part B occur at a high point of the graph. High points like these are the **maximum values** of the function. Low points of the graph are called **minimum values** of the function. What is the minimum value of the function shown? At what  $x$ -values does the minimum occur? (Hint: You should find at least three minimum values and the associated  $x$ -values.)

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Part D: How far apart do the maximum values occur? How far apart do the minimum values occur? Determining the distance between the  $x$  – values of the maximum values (or the  $x$  – values of the minimum values) is an easy way to determine the **period** of a function.

## Formalizing the concept of period:

Because we can say that  $f(1) = f(5) = f(9)$ , and so on, we could generalize by saying that  $f(x) = f(x + 4)$  for all  $x$ . That is, the function values repeat every four units. Of course, they also repeat every eight units and every 12 units. However, to list the period, we always choose the smallest interval. In general, the period of a function is the smallest positive number  $P$  such that  $f(x) = f(x + P)$  for all  $x$ .

Part E: The function is increasing on the interval  $(2, 5)$ . List two other intervals over which the function is increasing.

Part F: The function is decreasing on the interval  $(1, 2)$ . List two other intervals over which the function is decreasing.

8) Consider the function described in the following table. Label each of the statements below as True or False.

$x$	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
$f(x)$	7.2	5.1	4.0	4.7	6.8	8.9	10.0	9.3	7.2	5.1	4.0	4.7	6.8	8.9	10.0	9.3	7.2	5.1

Part A: The function appears to be periodic.

Part B: The period is 7.

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Part C: Assuming the pattern continues, then we would expect  $f(20) = 4.7$ .

Part D: The function is decreasing on the interval  $(4,8)$ .

Part E: Assuming the pattern continues, we would expect the function to be increasing on the interval  $(18,22)$ .

- 9) Write down one or two questions you want to ask in class in order to better understand periodic functions.

## Monitoring your readiness

- 9) To effectively plan and use your time wisely, it helps to think about what you know and do not know. How confident are you that you can:

Part A: Calculate the percent change between two values?

Part B: Write a formula for an exponential function given its starting value and percent rate of change?

Part C: Relate the base of an exponential function to its percent rate of change?

Part D: Use interval notation?

Part E: Identify the period of a periodic function?

Part F: Identify whether or when a function is increasing or decreasing?

If you are not confident in the above skills, you should seek help by:

- seeing your instructor before class,
- asking your instructor for on-campus resources,
- setting up a study group with classmates, or
- working with a tutor.